

Asignatura : Cálculo Diferencial , PMM 1137

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1. Preliminares:

$$a) f(x) = c$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{c - c}{h} = \lim_{h \rightarrow 0} 0 = 0$$

$$b) f(x) = mx + b$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{m(x+h) - mx - b}{h}$$

$$= \lim_{h \rightarrow 0} \frac{mx + mh + b - mx - b}{h}$$

$$= \lim_{h \rightarrow 0} \frac{mh}{h} =$$

$$= \lim_{h \rightarrow 0} m$$

$$= m$$

$$c) f(x) = ax^2 + bx + c$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{a(x+h)^2 + b(x+h) + c - ax^2 - bx - c}{h}$$

$$= \lim_{h \rightarrow 0} \frac{ax^2 + 2axh + ah^2 + bx + bh + c - ax^2 - bx - c}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2axh + ah^2 + bh}{h}$$

$$= \lim_{h \rightarrow 0} (2ax + ah + b)$$

$$= 2ax + b$$

$$d) f(x) = ax^3 + bx^2 + cx + d$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{a(x+h)^3 + b(x+h)^2 + c(x+h) + d - ax^3 - bx^2 - cx - d}{h}$$

$$= \lim_{h \rightarrow 0} \frac{ax^3 + 3ax^2h + 3axh^2 + ah^3 + bx^2 + 2bxh + bh^2 + cx + ch + d - ax^3 - bx^2 - cx - d}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3ax^2h + 3axh^2 + ah^3 + 2bxh + bh^2 + ch}{h}$$

$$= \lim_{h \rightarrow 0} (3ax^2 + 3axh + ah^2 + 2bx + bh + c)$$

$$= 3ax^2 + 2bx + c$$

2. Varios :

$$a) f(x) = \frac{2}{3}x^2$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & = \lim_{h \rightarrow 0} \frac{\frac{2}{3}(x+h)^2 - \frac{2}{3}x^2}{h} \\ & = \lim_{h \rightarrow 0} \frac{\frac{2}{3}x^2 + \frac{4}{3}xh + \frac{2}{3}h^2 - \frac{2}{3}x^2}{h} \\ & = \lim_{h \rightarrow 0} \frac{\frac{4}{3}xh + \frac{2}{3}h^2}{h} \\ & = \lim_{h \rightarrow 0} \left(\frac{4}{3}x + \frac{2}{3}h \right) \\ & = \frac{4}{3}x \end{aligned}$$

$$b) f(x) = 3x^3 + 2x^2 + 1$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & = \lim_{h \rightarrow 0} \frac{3(x+h)^3 + 2(x+h)^2 + 1 - 3x^3 - 2x^2 - 1}{h} \\ & = \lim_{h \rightarrow 0} \frac{3x^3 + 9x^2h + 9xh^2 + 3h^3 + 2x^2 + 4xh + 2h^2 + 1 - 3x^3 - 2x^2 - 1}{h} \\ & = \lim_{h \rightarrow 0} \frac{9x^2h + 9xh^2 + 3h^3 + 4xh + 2h^2}{h} \\ & = \lim_{h \rightarrow 0} (9x^2 + 9xh + 3h^2 + 4x + 2h) \\ & = 9x^2 + 4x \end{aligned}$$

$$c) f(x) = \frac{x-3}{x-4}$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & = \lim_{h \rightarrow 0} \frac{\frac{x+h-3}{x+h-4} - \frac{x-3}{x-4}}{h} \\ & = \lim_{h \rightarrow 0} \frac{(x+h-1)(x-2) - (x-1)(x+h-2)}{h(x-2)(x+h-2)} \\ & = \lim_{h \rightarrow 0} \frac{x^2 - 2x + hx - 2h - x + 2 - (x^2 + xh - 2x - x - h + 2)}{h(x+h-2)(x-2)} \\ & = \lim_{h \rightarrow 0} \frac{x^2 - 2x + hx - 2h - x + 2 - x^2 - xh + 2x + x + h - 2}{h(x+h-2)(x-2)} \\ & = \lim_{h \rightarrow 0} \frac{-h}{h(x+h-2)(x-2)} \\ & = \lim_{h \rightarrow 0} \frac{-1}{(x+h-2)(x-2)} \\ & = \frac{-1}{(x-2)^2} \end{aligned}$$

$$d) f(x) = \sqrt{2x-5}$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & = \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)-5} - \sqrt{2x-5}}{h} \cdot \frac{\sqrt{2(x+h)-5} + \sqrt{2x-5}}{\sqrt{2(x+h)-5} + \sqrt{2x-5}} \end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{2(x+h) - 5 - 2x + 5}{h(\sqrt{2(x+h)-5} + \sqrt{2x-5})} \\
&= \lim_{h \rightarrow 0} \frac{2x + 2h - 5 - 2x + 5}{h(\sqrt{2(x+h)-5} + \sqrt{2x-5})} \\
&= \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{2(x+h)-5} + \sqrt{2x-5})} \\
&= \lim_{h \rightarrow 0} \frac{2}{\sqrt{2(x+h)-5} + \sqrt{2x-5}} \\
&= \frac{2}{\sqrt{2x-5} + \sqrt{2x-5}} \\
&= \frac{1}{\sqrt{2x-5}}
\end{aligned}$$

e) $f(x) = \frac{1}{\sqrt{x+1}}$

$$\begin{aligned}
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\
&\lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h(\sqrt{x+h+1})(\sqrt{x+1})} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h(\sqrt{x+h})(\sqrt{x+1})} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\
&= \lim_{h \rightarrow 0} \frac{x - x - h}{h(\sqrt{x+h+1})(\sqrt{x+1})(\sqrt{x} + \sqrt{x+h})} \\
&= \lim_{h \rightarrow 0} \frac{-h}{h(\sqrt{x+h+1})(\sqrt{x+1})(\sqrt{x} + \sqrt{x+h})} \\
&= \lim_{h \rightarrow 0} \frac{-1}{(\sqrt{x+1})(\sqrt{x+1})(\sqrt{x} + \sqrt{x+h})} \\
&= \frac{-1}{(\sqrt{x+1})^2(2\sqrt{x})}
\end{aligned}$$

f) $f(x) = \sqrt[3]{x}$

$$\begin{aligned}
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&\lim_{h \rightarrow 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \cdot \frac{\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2}}{\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2}} \\
&= \lim_{h \rightarrow 0} \frac{x + h - x}{h[\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2}]} \\
&= \lim_{h \rightarrow 0} \frac{1}{h(\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2})} \\
&= \frac{1}{\sqrt[3]{x^2} + \sqrt[3]{x(x)} + \sqrt[3]{x^2}} \\
&= \frac{1}{3\sqrt[3]{x^2}}
\end{aligned}$$

g) $f(x) = \operatorname{sen} x$

$$\begin{aligned}
& \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sin x \cosh - \sin x + \cos x \sinh}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sin x (\cosh - 1) + \cos x \sinh}{h} \\
&= \lim_{h \rightarrow 0} \left(\frac{\sin x (\cosh - 1)}{h} + \frac{\cos x \sinh}{h} \right) \\
&= \lim_{h \rightarrow 0} \left(\sin x \frac{\cosh - 1}{h} + \cos x \frac{\sinh}{h} \right) \\
&= \lim_{h \rightarrow 0} \left(\sin x \frac{\cosh - 1}{h} \right) + \lim_{h \rightarrow 0} \left(\cos x \frac{\sinh}{h} \right) \\
&= \sin x \lim_{h \rightarrow 0} \left(\frac{\cosh - 1}{h} \right) + \cos x \lim_{h \rightarrow 0} \left(\frac{\sinh}{h} \right) \\
&= \sin x \lim_{h \rightarrow 0} \left(\frac{\cosh - 1}{h} \right) + \cos x \lim_{h \rightarrow 0} \left(\frac{\sinh}{h} \right) \\
&= \sin x \cdot 0 + \cos x \cdot 1 \\
&= \cos x
\end{aligned}$$

h) $f(x) = \cos x$

$$\begin{aligned}
& \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos x \cosh - \cos x - \sin x \sinh}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos x (\cosh - 1) - \sin x \sinh}{h} \\
&= \lim_{h \rightarrow 0} \left(\frac{\cos x (\cosh - 1)}{h} - \frac{\sin x \sinh}{h} \right) \\
&= \lim_{h \rightarrow 0} \left(\cos x \frac{\cosh - 1}{h} - \sin x \frac{\sinh}{h} \right) \\
&= \lim_{h \rightarrow 0} \left(\cos x \frac{\cosh - 1}{h} \right) - \lim_{h \rightarrow 0} \left(\sin x \frac{\sinh}{h} \right) \\
&= \cos x \lim_{h \rightarrow 0} \left(\frac{\cosh - 1}{h} \right) - \sin x \lim_{h \rightarrow 0} \left(\frac{\sinh}{h} \right) \\
&= \cos x \lim_{h \rightarrow 0} \left(\frac{\cosh - 1}{h} \right) - \sin x \lim_{h \rightarrow 0} \left(\frac{\sinh}{h} \right) \\
&= \cos x \cdot 0 - \sin x \cdot 1 \\
&= -\sin x
\end{aligned}$$

i) $f(x) = \tan x$

$$\begin{aligned}
& \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\
& \lim_{h \rightarrow 0} \frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos x \sin(x+h) - \sin x \cos(x+h)}{h(\cos(x+h)\cos x)}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{\cos(x(\cos x \cosh + \cos x \sinh) - \sin(x(\cos x \cosh - \sin x \sinh))}{h(\cos(x+h)\cos x)} \\
&= \lim_{h \rightarrow 0} \frac{\cos x \sin x \cosh + \cos^2 x \sinh - \sin x \cos x \cosh + \sin^2 x \sinh}{h(\cos(x+h)\cos x)} \\
&= \lim_{h \rightarrow 0} \frac{\cos^2 x \sinh + \sin^2 x \sinh}{h(\cos(x+h)\cos x)} \\
&= \lim_{h \rightarrow 0} \frac{\cos^2 x \sinh + \sin^2 x \sinh}{h(\cos(x+h)\cos x)} \\
&= \lim_{h \rightarrow 0} \frac{\sinh(\cos^2 x + \sin^2 x)}{h(\cos(x+h)\cos x)} \\
&= \lim_{h \rightarrow 0} \left[\frac{\sinh}{h} \cdot \frac{1}{\cos(x+h)\cos x} \right] \\
&= 1 \cdot \frac{1}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} \\
&= \sec^2 x
\end{aligned}$$

j) $f(x) = \sec x$

$$\begin{aligned}
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\
&\lim_{h \rightarrow 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos x - \cos(x+h)}{h(\cos(x+h)\cos x)} \\
&= \lim_{h \rightarrow 0} \frac{\cos x(\sin x \cosh + \cos x \sinh) - \sin x(\cos x \cosh - \sin x \sinh)}{h(\cos(x+h)\cos x)} \\
&= \lim_{h \rightarrow 0} \left[\frac{1}{\cos(x+h)\cos x} \cdot \frac{\cos x - \cos(x+h)}{h} \right] \\
&= \lim_{h \rightarrow 0} \left[\frac{1}{\cos(x+h)\cos x} \cdot \frac{\cos x - \cos x \cosh + \sin x \sinh}{h} \right] \\
&= \lim_{h \rightarrow 0} \left[\frac{1}{\cos(x+h)\cos x} \cdot [\cos x \frac{1 - \cosh}{h} + \sin x \frac{\sinh}{h}] \right] \\
&= \frac{\sin x}{\cos x \cos x} \\
&= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \\
&= \tan x \sec x
\end{aligned}$$

k) $f(x) = \sqrt{\cos x}$

$$\begin{aligned}
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\
&\lim_{h \rightarrow 0} \frac{\sqrt{\cos(x+h)} - \sqrt{\cos x}}{h} \cdot \frac{\sqrt{\cos(x+h)} + \sqrt{\cos x}}{\sqrt{\cos(x+h)} + \sqrt{\cos x}} \\
&= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h(\sqrt{\cos(x+h)} + \sqrt{\cos x})} \\
&= \lim_{h \rightarrow 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h(\sqrt{\cos(x+h)} + \sqrt{\cos x})} \\
&= \lim_{h \rightarrow 0} \frac{\cos x(\cosh - 1) - \sin x \sinh}{h} \cdot \frac{1}{\sqrt{\cos(x+h)} + \sqrt{\cos x}}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} [\cos x \frac{\cosh - 1}{h} - \sin x \frac{\operatorname{senh}}{h} \cdot \frac{1}{\sqrt{\cos(x+h)} + \sqrt{\cos x}}] \\
&= \frac{-\sin x}{2\sqrt{\cos x}} \\
l) \quad &f(x) = \sqrt{\sin x} \\
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\
&\lim_{h \rightarrow 0} \frac{\sqrt{\sin(x+h)} - \sqrt{\sin x}}{h} \cdot \frac{\sqrt{\sin(x+h)} + \sqrt{\sin x}}{\sqrt{\sin(x+h)} + \sqrt{\sin x}} \\
&= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h(\sqrt{\sin(x+h)} + \sqrt{\sin x})} \\
&= \lim_{h \rightarrow 0} \frac{\sin x \cosh + \cos x \operatorname{senh} - \sin x}{h(\sqrt{\sin(x+h)} + \sqrt{\sin x})} \\
&= \lim_{h \rightarrow 0} \frac{\sin x (\cosh - 1) + \cos x \operatorname{senh}}{h} \cdot \frac{1}{\sqrt{\sin(x+h)} + \sqrt{\sin x}} \\
&= \lim_{h \rightarrow 0} [\sin x \frac{\cosh - 1}{h} + \cos x \frac{\operatorname{senh}}{h} \cdot \frac{1}{\sqrt{\sin(x+h)} + \sqrt{\sin x}}] \\
&= \frac{\cos x}{2\sqrt{\sin x}}
\end{aligned}$$

m) $f(x) = |x|$ Podemos redefinir f de la siguiente manera

$$f(x) = \begin{cases} x & \text{si } x \geq 0; \\ -x & \text{si } x < 0. \end{cases}$$

En el límite $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ vamos a hacer la evaluación en $x = c$ quedando $\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$

Haciendo la sustitución $x = c + h$ tenemos que si $h \rightarrow 0$ entonces $x \rightarrow c$.

$$\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

Haciendo análisis de límites laterales tenemos que :

$$\begin{aligned}
&\lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \\
&\lim_{x \rightarrow 0^+} \frac{x - 0}{x - 0} = \\
&\lim_{x \rightarrow 0^+} 1 = 1 \\
&\lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \\
&\lim_{x \rightarrow 0^-} \frac{-x - 0}{x - 0} = \\
&\lim_{x \rightarrow 0^-} -1 = -1
\end{aligned}$$

Como los límites laterales son distintos se concluye que el límite dado no existe para $x = 0$

3. Relevante $f(x) = x^n$ para $n = 1, 2, 3, 4, \dots$

Para $n = 1$, $f(x) = x$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & \lim_{h \rightarrow 0} \frac{x+h-x}{h} \\ & = \lim_{h \rightarrow 0} \frac{h}{h} \\ & = 1 \end{aligned}$$

Para $n = 2$, $f(x) = x^2$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ & = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ & = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ & = \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} \\ & = \lim_{h \rightarrow 0} (2x+h) \\ & = 2x \end{aligned}$$

Para $n = 3$, $f(x) = x^3$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ & = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ & = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ & = \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} \\ & = \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) \\ & = 3x^2 \end{aligned}$$

Para n , $f(x) = x^n$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\ & \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\ & = \lim_{h \rightarrow 0} \frac{\sum_{k=0}^n \binom{n}{k} a^{n-k} b^k - x^n}{h} \end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{\binom{n}{0}x^n + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \dots + \binom{n}{n}h^n - x^n}{h} \\
&= \lim_{h \rightarrow 0} \frac{x^n + nx^{n-1}h + \binom{n}{2}h^2 + \dots + h^n - x^n}{h} \\
&= \lim_{h \rightarrow 0} \frac{nx^{n-1}h + \binom{n}{2}h^2 + \dots + h^n}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(nx^{n-1} + \binom{n}{2}h + \dots + h^{n-1})}{h} \\
&= \lim_{h \rightarrow 0} (nx^{n-1} + \binom{n}{2}h + \dots + h^{n-1}) \\
&= nx^{n-1}
\end{aligned}$$

4. Práctica

$$\begin{aligned}
a) \quad &f(x) = 2x^3 + 4x^2 + 10 \\
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\
&\lim_{h \rightarrow 0} \frac{2(x+h)^3 + 4(x+h)^2 + 10 - 2x^3 - 4x^2 - 10}{h} \\
&= \lim_{h \rightarrow 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) + 4(x^2 + 2xh + h^2) + 10 - 2x^3 - 4x^2 - 10}{h} \\
&= \lim_{h \rightarrow 0} \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 + 4x^2 + 8xh + 4h^2 + 10 - 2x^3 - 4x^2 - 10}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(6x^2 + 6xh + 2h^2 + 8x + 4h)}{h} \\
&= \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2 + 8x + 4h) \\
&= 6x^2 + 8x \\
b) \quad &f(x) = -\sqrt{x-1} \\
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\
&\lim_{h \rightarrow 0} \frac{-\sqrt{x+h-1} + \sqrt{x-1}}{h} \\
&= -\lim_{h \rightarrow 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h} \cdot \frac{\sqrt{x+h-1} + \sqrt{x-1}}{\sqrt{x+h-1} + \sqrt{x-1}} \\
&= -\lim_{h \rightarrow 0} \frac{x+h-1-x+1}{h(\sqrt{x+h-1} + \sqrt{x-1})} \\
&= -\lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h-1} + \sqrt{x-1})} \\
&= \frac{1}{2\sqrt{x-1}} \\
c) \quad &f(x) = \frac{1}{x} \\
&\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =
\end{aligned}$$

$$\begin{aligned}& \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\&= \lim_{h \rightarrow 0} \frac{x - x - h}{hx(x+h)} \\&= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\&= \frac{-1}{x^2}\end{aligned}$$